

# Kite Flight Simulators Based on Minimal Coordinate Formulations



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## Introduction

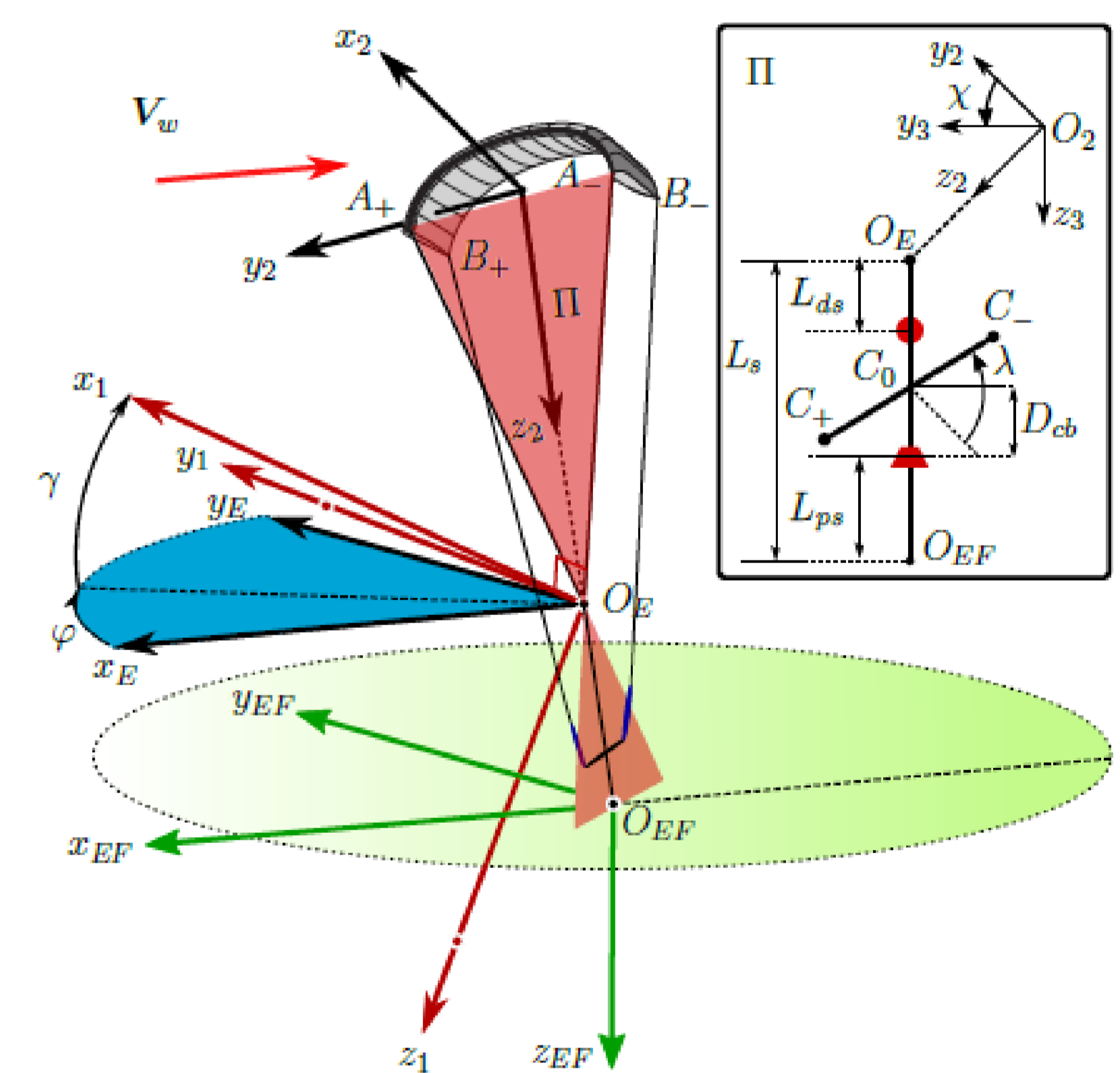
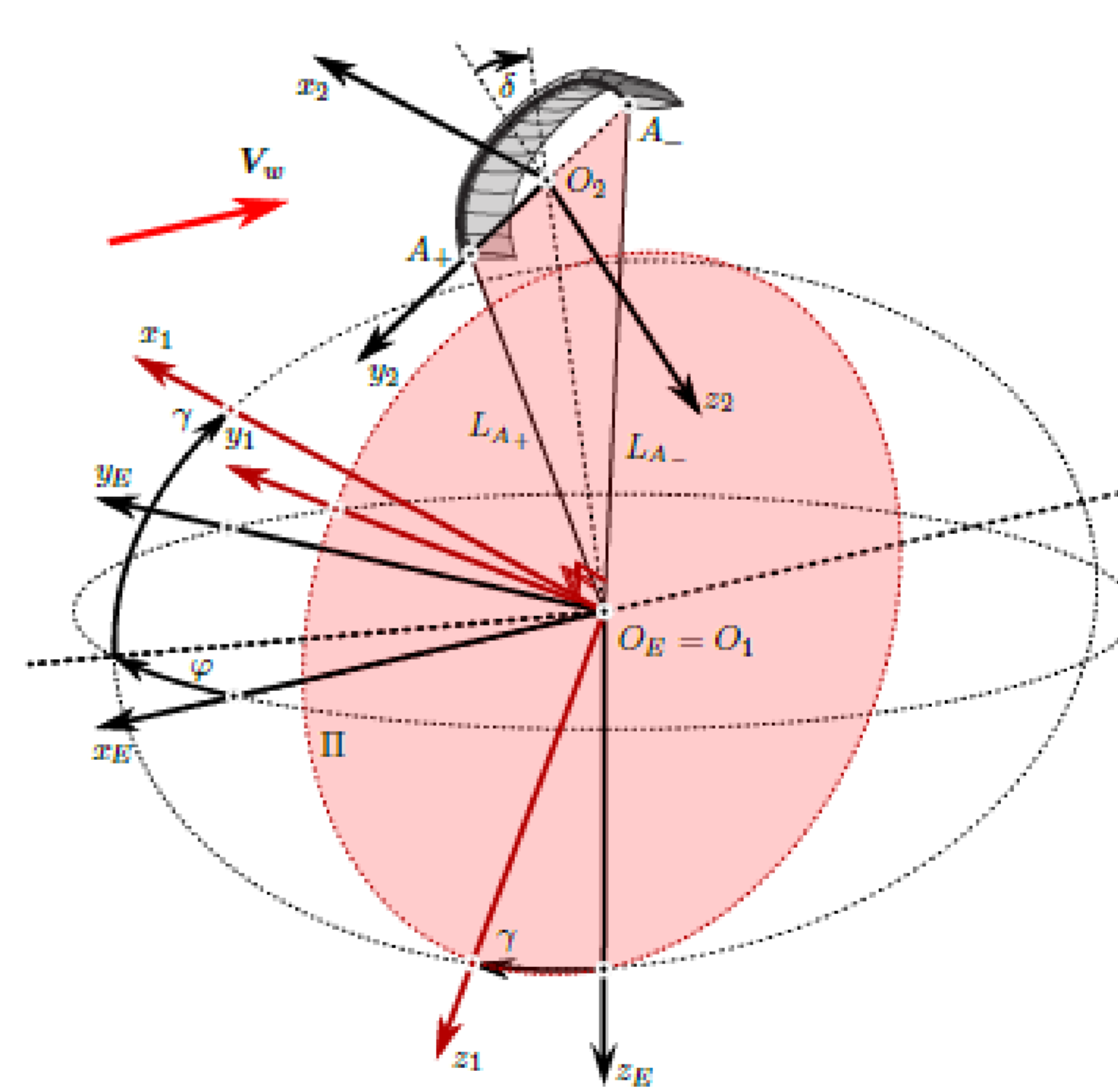
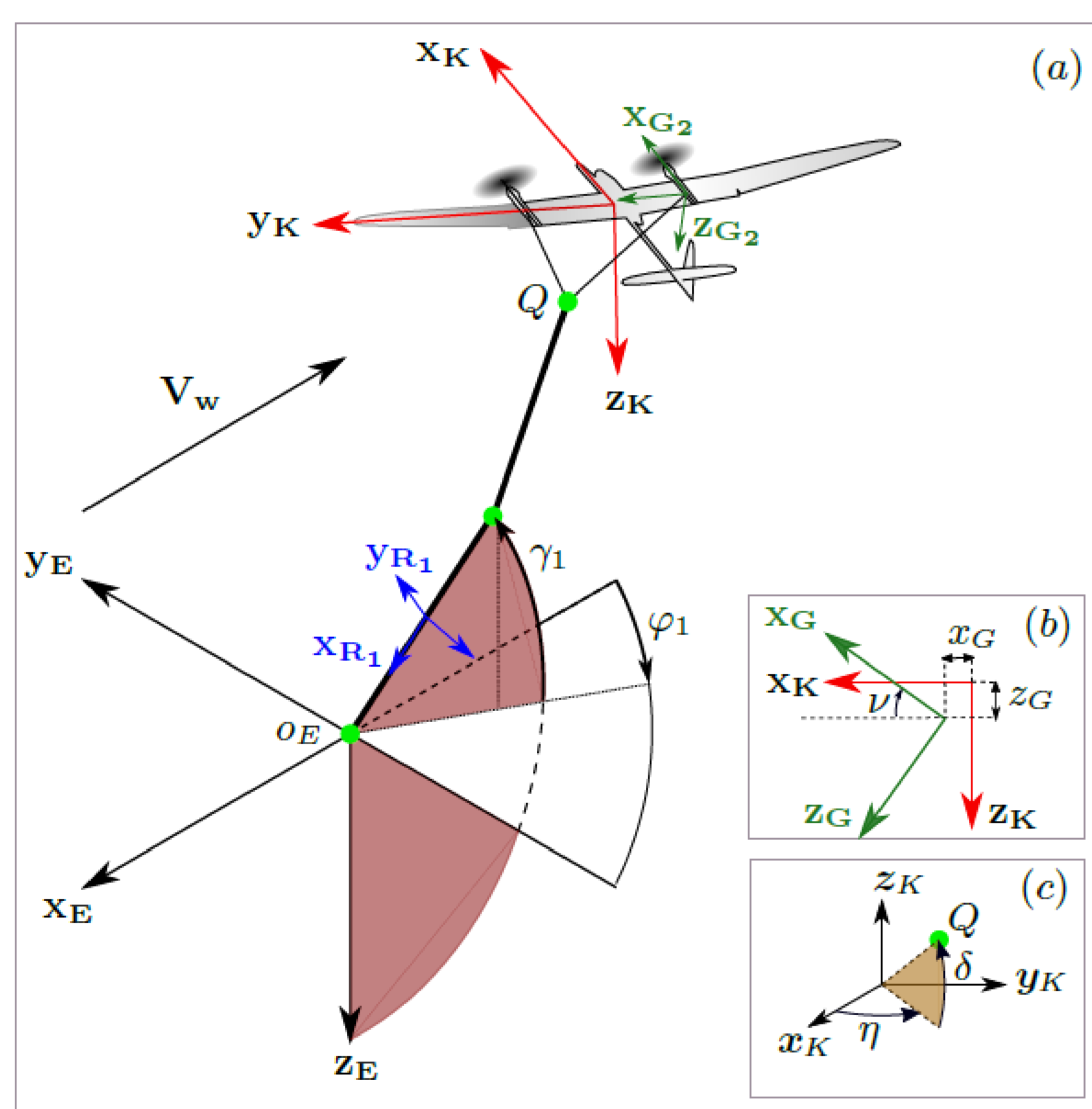
**Modelling tether dynamics: a complex task.** Flight simulators of Airborne Wind Energy Systems (AWEs) involve inevitably the modelling of one or several tethers. If elasticity is included, the typically high tether stiffness yields to the coexistence of very fast longitudinal and much slower transverse waves. The cost of the numerical integration is high because the resulting set of equations is stiff. Alternatively, one can substitute each tether by a collection of straight and inelastic bars, thus ignoring elasticity effects but keeping tether flexibility accurately for a large enough number of bars in the discretization. Although this methodology avoids the stiffness issue, because the fast longitudinal oscillations are automatically eliminated, it presents a drawback: classical formulation based on Newton's laws yields to a set of ordinary differential equations coupled to nonlinear algebraic equations due to the geometric constraints imposed by the inelastic tethers. The numerical cost is again increased because a Newton-Raphson scheme should be implemented each time step in order to find the constraint forces (tether tensions) that are consistent with the geometric constraints.

**Minimal coordinate formulation: an efficient solution.** Analytical mechanics (Lagrangian formulation) using a minimal coordinate formulation is an efficient way to write the equations of motion of inelastic tethers modelled by a set of straight bars. The geometric constraints are taken into account when the state variables are chosen and the dynamic of the system (Euler-Lagrange equations) is governed by a set of ordinary differential equation. Since they are not stiff and they are not coupled to nonlinear algebraic equations, this methodology yields to efficient and robust simulators. Although the Lagrangian formulation typically requires a higher analytical and intellectual effort, the simulators can be used as *black boxes* in more sophisticated analyses such as optimal control determination and the user does not need knowledges on analytical mechanics.

**LAKSA: a free numerical tool for AWEs analysis.** This work implemented a minimal coordinate approach to find the equations of motion of a kite with 1, 2 and 4 lines. They cover most of kites applications, including airborne wind energy generation (FlyGen and GroundGen schemes), traction uses, acrobatic kites analysis and kitesurf. The three simulators are integrated in a single tool named LAKSA (LAgrangian Kite SimulAtors) with very easy user interfaces. Once the code will be registered in few months, the Matlab codes and the manual will be freely available in the ResearchGate profile of the first author (find also there UC3M publications and projects on AWEs)

## LAKSA (Lagrangian Kite Simulator)

|                   | KiteFlex (1 Line)  | KiteAcrobat (2-Lines)                  | KiteSurf (4-Lines)                              |
|-------------------|--|--|---|
| Mechanical System | Rigid Kite<br>1 tether ( $N_r$ Bars), Bridle, $N_g$ generators | Rigid Kite<br>2 straight tethers       | Rigid Kite<br>2 inelastic and 2 elastic tethers |
| Language          | Matlab, Fortran (parallelized)                                 | Matlab                                 | Matlab  |
| Applications      | Fly-Generation, Ground-Generation                              | Traction Applications, Acrobatic Kites | Kitesurf  |
| Control           | Tether length, bridle, motor torque                            | Tether lengths                         | Control bar position                            |

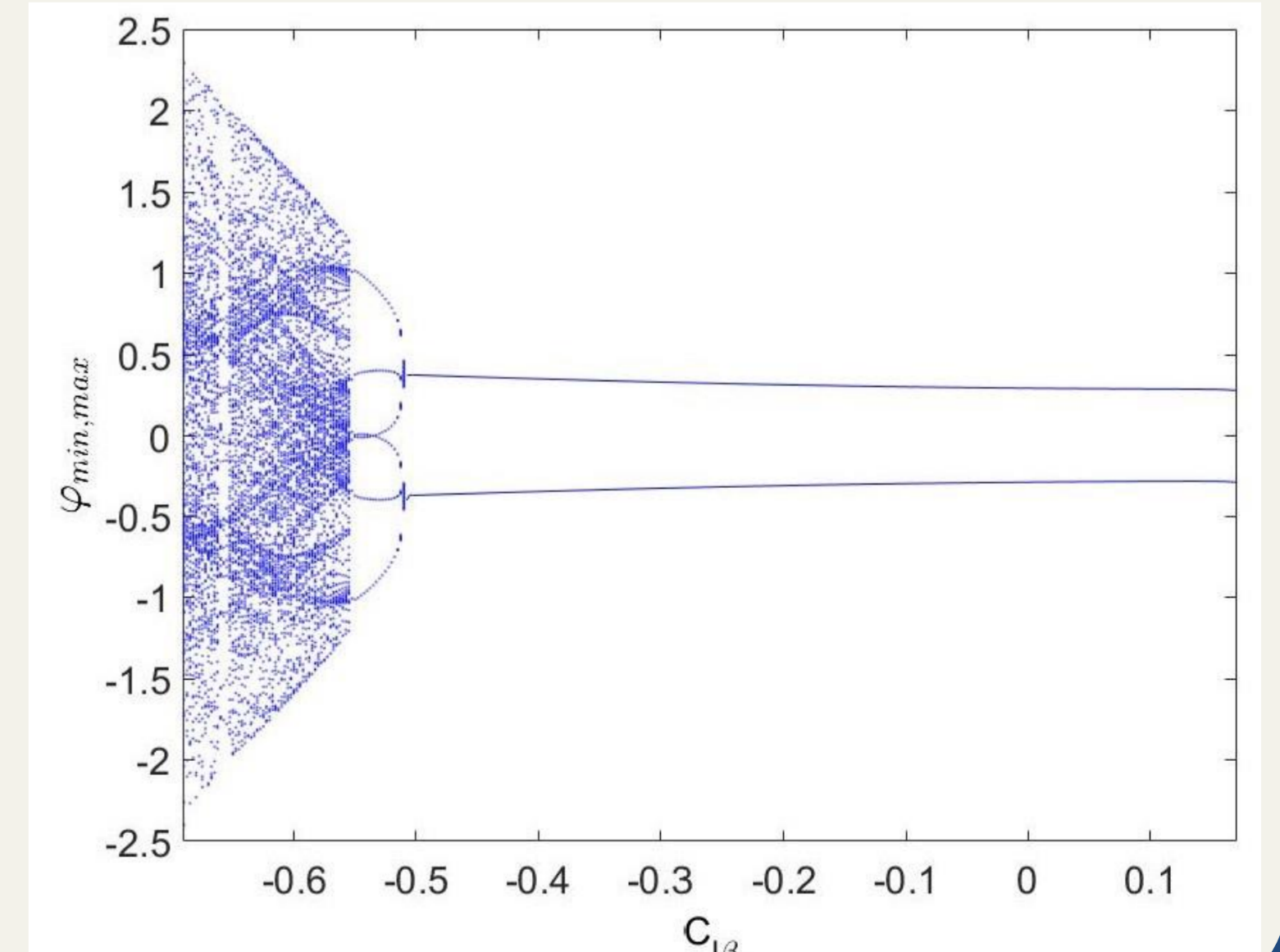
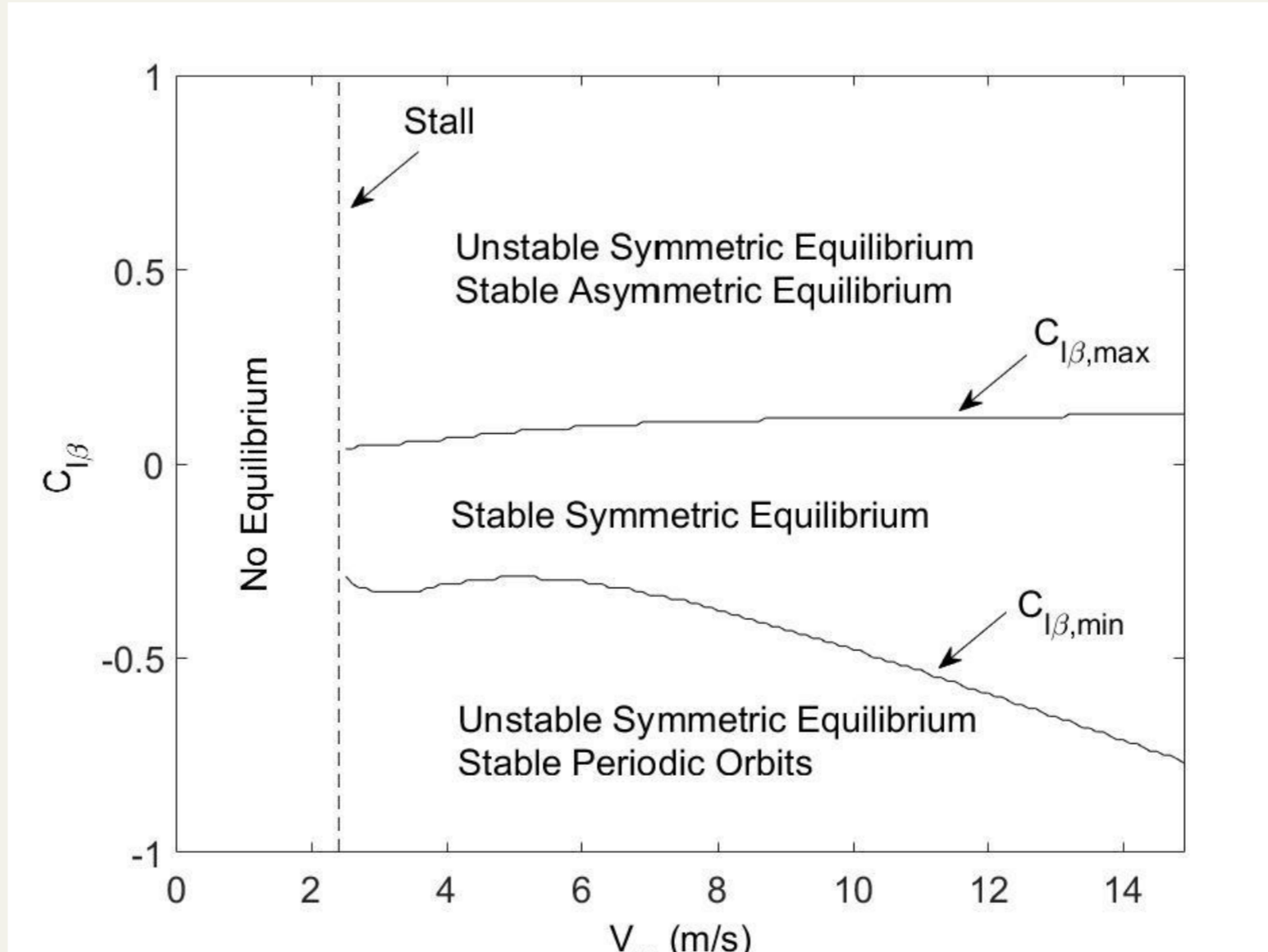
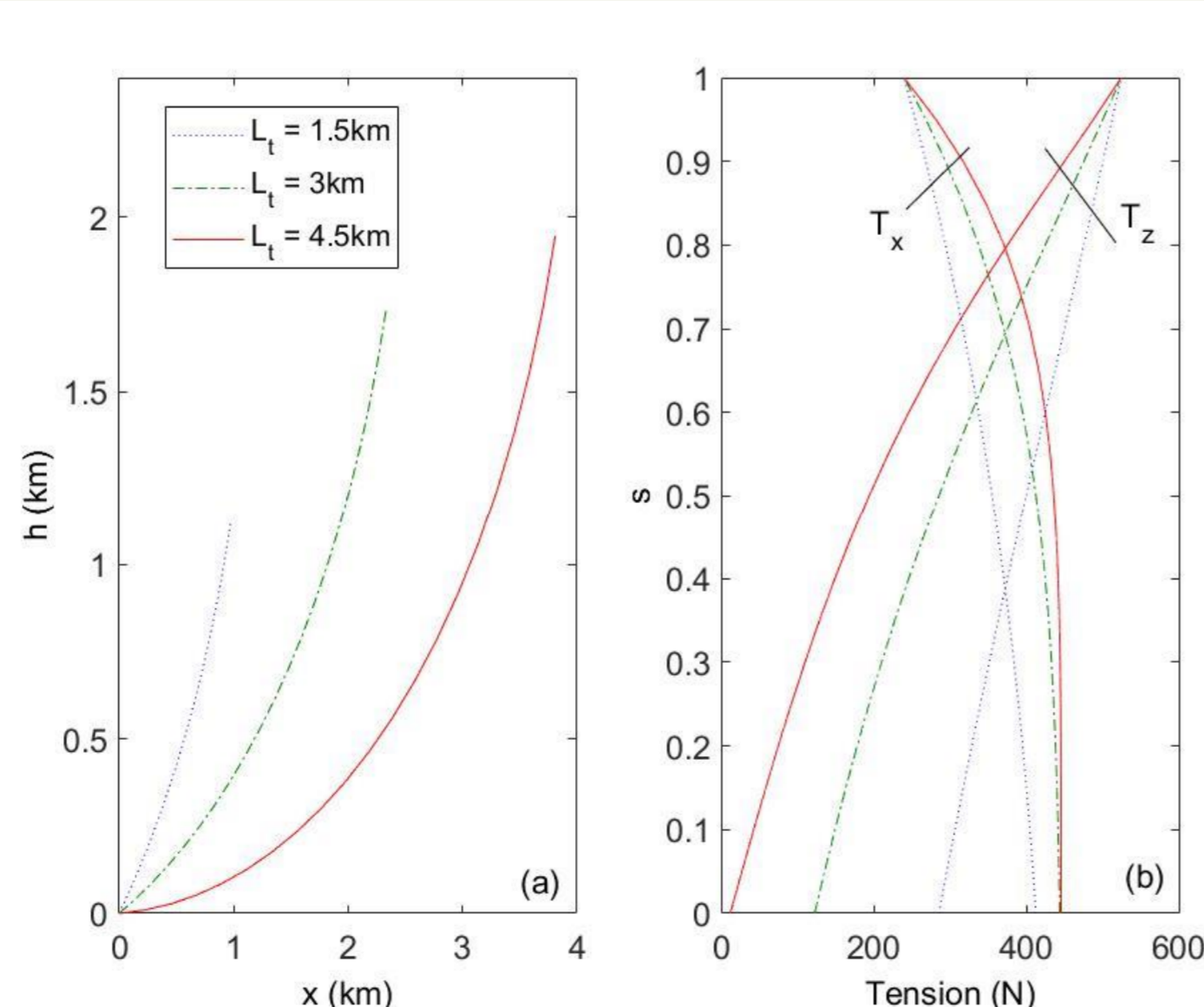


## Analysis of AWEs with LAKSA

The analysis of the dynamics of AWEs is simple with LAKSA: (i) the user defines the physical parameters, such as kite geometry (chord, span, surface), aerodynamic coefficients, tether properties (density, drag coefficient, diameter), bridle geometry, ambient parameters (wind velocity, gravitational acceleration), and the generator and blades characteristics, (ii) explicit time-dependent laws should be given for the control vector, including tether length, bridle geometry, and motor torques, (iii) the equations of motion are integrated numerically, (iv) the evolution of the state vector is post-processed to get the performance of the system such as the position, and velocity of the kite, the tether tensions, generated power, etc. Since modular, the code extension to implement other aerodynamic models, more complex wind velocity descriptions, or even close-loop control with path trackers, is straightforward.

### Some results with LAKSA

Left panel shows the tether and kite equilibrium with KiteFlex. Center panel displays the different domains in the wind velocity-dihedral effect plane of KiteAcrobat. The right panel is a bifurcation diagram (lateral angles displacement versus dihedral effect) of KiteAcrobat when the lengths of the two tethers are varied periodically.



Find more details on LAKSA codes and results in the following references :

- **KiteFlex:** [1] J. Alonso-Pardo and G. Sánchez-Arriaga, Kite Model with Bridle Control for Wind-Power Generation, *J. of Aircraft* 52, 3, 2015, [2] A. Pastor-Rodríguez, G. Sánchez-Arriaga, M. Sanjurjo-Rivo, Modeling and Stability Analysis of Tethered Kites altitudes at High Altitudes, *Journal of Guidance, Control, and Dynamics*, 40(8), 1892-1901, 2017. [3] A. Pastor-Rodríguez, G. Sánchez-Arriaga, M. Sanjurjo-Rivo, R. Schmehl, A kite flight simulator for the analysis of ground and on-board airborne wind energy generation systems (in preparation)
- **KiteAcrobat:** [4] G. Sánchez-Arriaga, M. García-Villalba, R. Schmehl, *Applied Mathematical Modelling*, 47, 473-486, 2017.
- **KiteSurf:** [5] R. Borobia, G. Sánchez-Arriaga, M. García-Villalba and R. Schmehl, Flight testing and dynamic simulation of four-line power kites (in preparation).

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