

# Inertia-supported pumping cycles based on a roto-kite

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## INTRODUCTION

Rotary kites show, like tethered multiple-kite systems, a great potential over single-kite systems due to the reduced main tether drag [1]. Fig.1 depicts both variants. Over tethered multiple-kites, rotary kites have the advantage of **simpler dynamics** and **minimal secondary tether drag**. On the downside, they have a higher **induction factor** and they scale badly due to the **small power-to-mass ratio**.

The research question of this poster entails the numerical computation of an optimal pumping cycle for a small-scale AWE system based on a rotary kite. The optimal cycle is compared to a classical pumping cycle based on Loyd's insights [2], in order to evaluate its theoretical and practical implications.

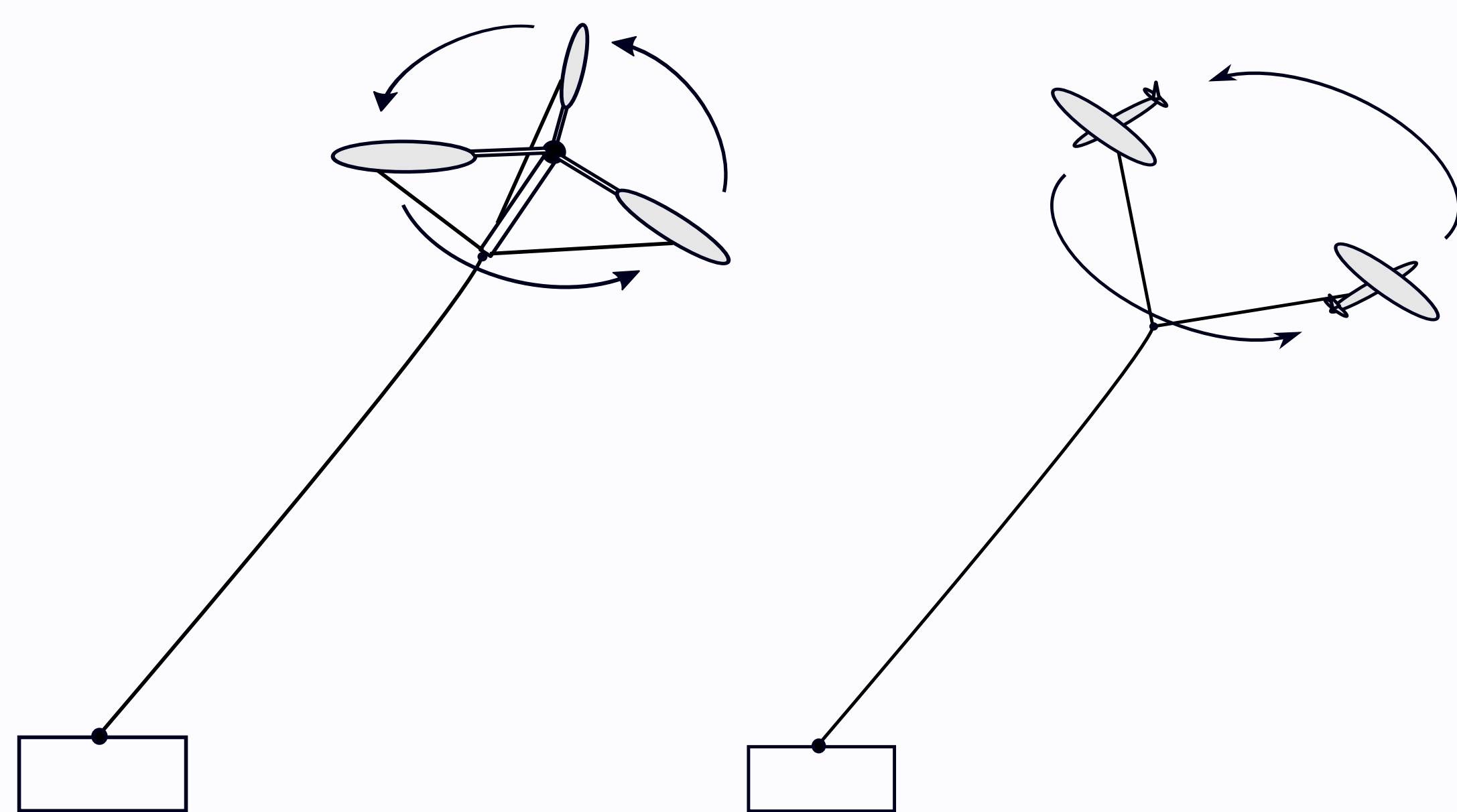


Figure 1: Roto-kite (left); Tethered multiple-kites (right)

## SYSTEM MODEL

The roto-kite is modeled in **natural coordinates**, and the  $SO(3)$  Lie group is parametrized by the **Direct Cosine Matrix**. These choices lead to simple symbolics, cheap function evaluations and reduce the non-linearity of the dynamics [3]. After index reduction, the dynamics are given in the form of an index-1 DAE. **Baumgarte stabilization** is performed on all invariants present in the model.

The aerodynamic model is based on **thin airfoil theory** with lifting-line theory for elliptical wings. Rotational and side-slip stability derivatives, and airfoil-airmass interaction are not accounted for. Tether elasticity and sagging are not modeled.

The system is controlled by **cyclic pitch control** and the **tether jerk** is assumed to be controlled directly.

## PERIODIC OPTIMAL CONTROL

**Optimal power cycles** are found by solving the periodic optimal control problem (POCP):

$$\begin{aligned} & \text{minimize} && -\frac{1}{T}E(T) + \int_0^T u(t)^\top R u(t) dt \\ & x(\cdot), \dot{x}(\cdot), u(\cdot), z(\cdot), \theta, T && \\ & \text{subject to} && F(x(t), \dot{x}(t), u(t), z(t), \theta, T) = 0, \\ & && x(0) - x(T) = 0, \\ & && h(x(t), \dot{x}(t), u(t), z(t), \theta, T) \geq 0. \end{aligned}$$

The classical **'Loyd' cycle** is obtained by additionally limiting the reel-out speed to a third of the average wind speed.

## RESULTS

Solving the POCP leads to **very short cycles**, and an optimal pumping cycle that differs greatly from the classical Loyd cycle. The results are summarized in Fig. 2 and Table 1.

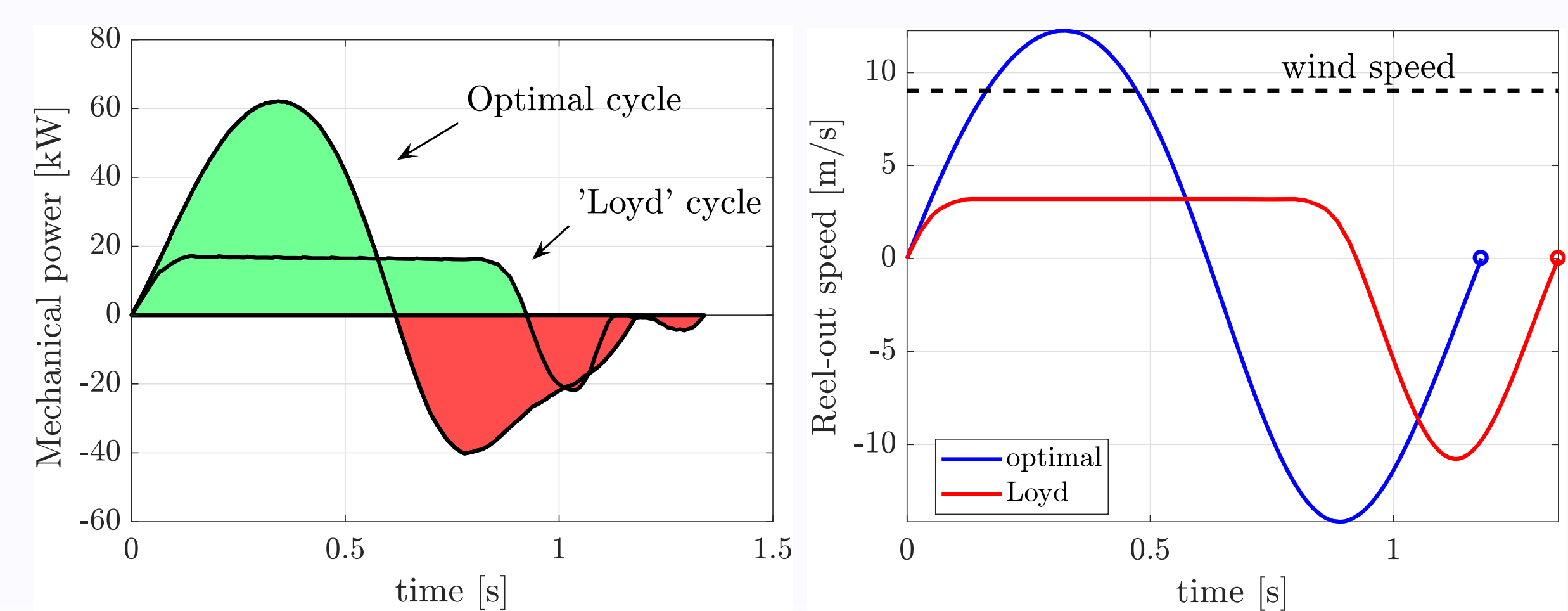


Figure 2: Comparison of optimal and classical-Loyd pumping cycles

	optimal	'Loyd'
Loyd factor	101.7%	88.5%

Table 1: Loyd factor of optimal and classical-Loyd pumping cycles

How can a pumping AWE system reach Loyd's limit? Answer: the roto-kite **continuously extracts energy** from the wind, storing it as kinetic energy during reel-in, as the roto-kite power balance, depicted in Fig. 3, shows.

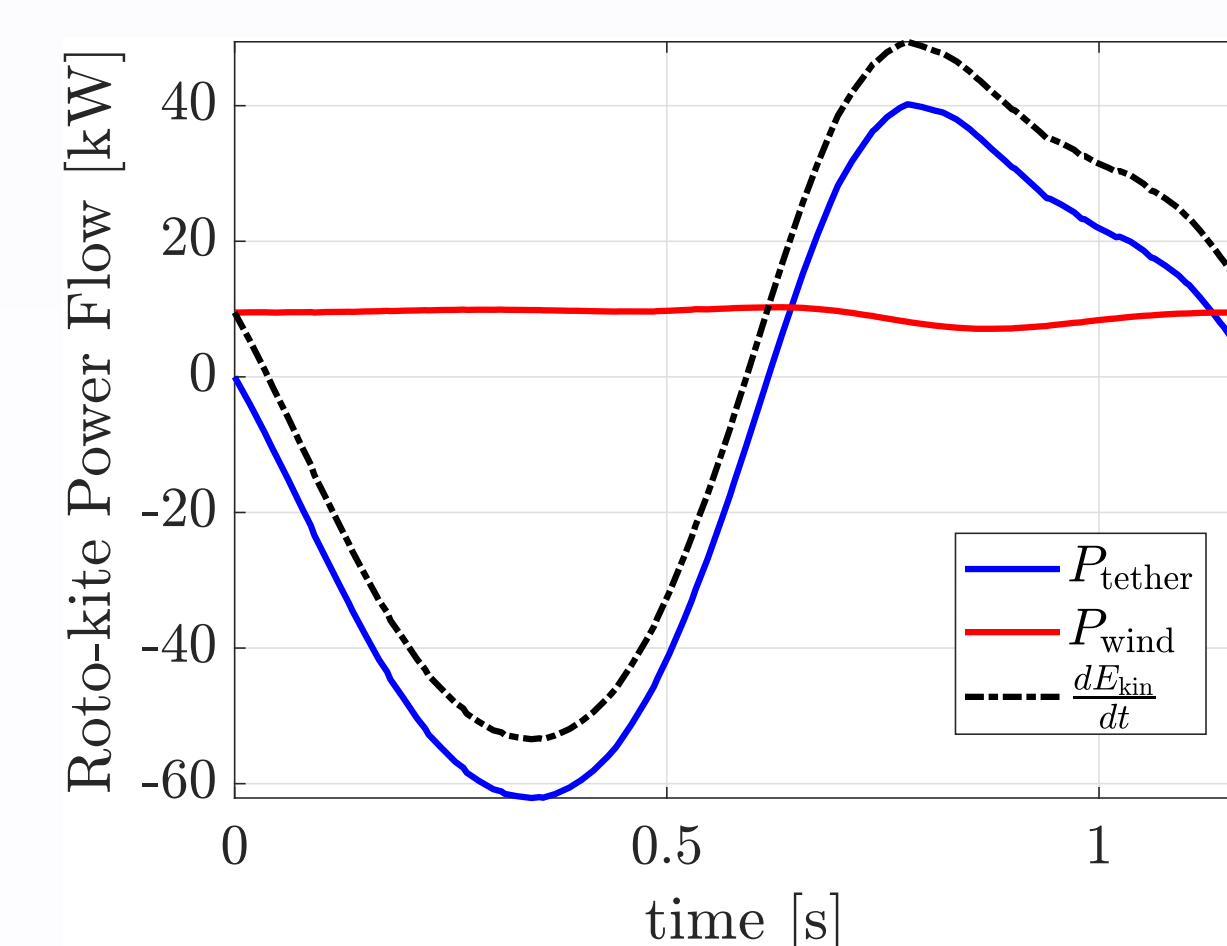


Figure 3: Roto-kite power balance

## CONCLUSION

The theoretical feasibility of a pumping cycle with a roto-kite has been shown. Supported by inertia, a small-scale system shows the theoretical potential to approximate Loyd's power limit. The quasi-sinusoidal, high-frequent reel-out profile could e.g. be realized by connecting the tether at the ground station to the rotor of a vertical-axis generator. Future research should entail investigating the influence of tether elasticity and the airfoil-airmass interaction.

## REFERENCES & ACKNOWLEDGEMENTS

- [1] Zanon, M., Gros, S., Andersson, J., Diehl, M., *Airborne Wind Energy Based on Dual Airfoils*, IEEE Transactions on Control Systems Technology, Vol. 21, 2013.
- [2] Loyd, M. L., *Crosswind Kite Power*, Journal of Energy, Vol. 4, No. 3, 1980, pp. 106-111
- [3] Gros, S., Diehl, M., *Modeling of Airborne Wind Energy Systems in Natural Coordinates*, Airborne Wind Energy, Springer, 2013

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